**The Applications of Partial Derivative**

1. **Traffic Flows**

* **Problem Statement**

A traffic congestion will bring various negative influences to a county. In Ghana, traffic congestion had caused a delay in its economic development. This is due to the lost productive time caused by the long traffic jams. Besides that, the high congestion level jeopardized the environment as the poisonous gasses were emitted into the air. According to Addison (2016), some countries in Ghana were even suffering traffic congestion for three to eight hours daily.

Thus, partial differential Equations (PDEs) are used based on the Lighthill and Whitham (1955) and Richard (1956) (LWR) model to manage the vehicle traffic flow between the traffic lights in the cities of Ghana (from Kwame Nkrumah Circle to Adabraka) effectively. Furthermore, the method of characteristics is also used to solve the systems of PDEs.

* **Methodology**

In the thesis, the traffic flow is discussed by using a macroscopic approach which is based on the LWR model. It shows the fundamental relationship among the three traffic flow parameters (density, flow, and speed) as below:

where *q* is flow (*veh/s*), *v* is speed (*m/s*), and *k* is density (*veh/m*).

According to Greenshields (1935), the relationship between *v* and *k* can be written by

(1)

where *vmax* is some maximal velocity as a property of the road, and *kmax* is the maximal bumper to bumper density.

When multiplying both sides of equation (1) with *k*,

(2)

Based on the LWR model, the traffic flow can be written in a one-dimensional continuity equation

where *k(x, t)* is the density (in number of vehicles per unit length of a road at time *t*), and *v* is the traffic stream velocity.

After doing partial equation, the LWR model can be expressed as

(3)

* **Results and Discussion**

The data was collected and analyzed. The equation (2) is then used to show the result of regression for the flow versus density:

(4)

* **Non-linear homogeneous PDE**

From equation (4), , the maximum point can be found.

By substituting into ,

while the velocity at the maximum flow is

From the LWR model (equation (3)),

(5)

Since the solution is *k = k(x, t)*, taken the derivative with respect to *t*,

(6)

Compare equation (5) with equation (6),

So, on a certain curve *x(t)* the solution *k(x, t)* of equation (4) is a constant.

* **Non-linear inhomogeneous PDE**

Let ,

When integrated,

When ,

Hence,

When , then

where *k* is density, *β* is a constant and *t* is time. This represents the particular solution of the general solution.

* **Method of Characteristic**

Apply the method of characteristics to

along the initial condition *k(x, 0) = 93.87x* at *t = 0*.

Thus, making *x* the subject in

as the characteristic which starts at *x = x0* when *t = 0*.

Substituting *k(x0, 0) = 93.87 x0* into the above equation

The solution of non-linear homogeneous is obtained by making *x0* the subject of the characteristic equation. Thus

Since *k(x, t) = F(x0)*, the solution of the non-linear homogeneous PDE is given as

where *k(x, t)* is the density (in number of vehicles per unit length of a road at time *t*), *x* is the position and *t* is time. This represents the complementary function of the general solution.

* **Results and Discussion**

The maximum flow on the road network was *0.511 veh/s*.

The maximum density was *53.505 veh/m*.

The maximum density was *0.0096 m/s*.

To prevent traffic congestion, traffic flow should be designed to move at a density corresponding to maximum traffic flow.

1. **Economics and Business**

* **Problem Statement**

Nowadays, cash flow conversions and expanding the business become the focus for every business. The article “Using Partial Differential Equations for Pricing of Goods and Services” discuss the methodology of comparative analysis by using an approach for pricing various goods and services. The objective of this study is to investigate the possibility of profit from sales of services and goods. This study also able to show the best possible prices in order to maximize the profits.

* **Methodology**

This research was conducted by using partial differential equation. This can be used for pricing of goods on option. Black-Scholes model was solved by using final differential methods.

1. Final differential methods for Black-Scholes equation

*S(t)* represent price

*t* represent the time

*f(S, t)* is a function used to calculate value of the underlying company asset.

+ *rS*+= *rf*  (1)

In equation (1), appropriate boundary condition was used to characterize the type of Option. We need to solve this equation by a method called finite differential method.

Let *T* be the maturity of Option and  suitable clear and bright asset prices *S(t)*. The field of PDE is limitless in term of asset prices so we need use the term .The network consists the points *(S, t)*, such that:

S=0, ,



Annotation for the grid: .

There are some ways used to approximate the partial derivatives of equation (1):

* Progressive difference
* Backward difference
* Backward difference
* Second derivative

We used to different approaches in this experiment which are explicit or implicit. Definition of boundary condition also have to take into account. For price K the boundary condition is:



1. Pricing for European Option with explicit method

From equation (1), we only consider the European Option but it also need compatible with the boundary condition. As a result, we obtain another equation:

(2)

Where

In equation (2), let *j*=*N*.

The terminal condition have unknown valueexpressed as a function of the three known values. We obtain the explicit scheme from equation (2)

(3)

where

1. Pricing for European Option with implicit method

We use implicit method because there is some numerical instability occur in explicit method. We obtain the following equation for the grid:

(4)

where, for each *i*,

* **Results and Discussion**

The function blsprice used to calculate the pricing of European Option with the right to buy and sell (European put and call Option) by using Black-Scholes model.

[Call, Put] = blsprice (Price, Strike, Rate, Time, Volatility)

where:

Price = current price of the underlying asset

Strike = The price of Option

Rate = The annual interest rate

Time = Duration of Option in years

Volatility = The annual instability of assets

We obtain 3.9663 for European Put Option (Put = 3.9663) by using the function blsprice.

[Call, Put] = blsprice (60, 60, 0.2, 6/12, 0.4)

We obtain by explicit scheme with the same values for corresponding parameter is 3.8993.

PricingEurOptExpl (60, 60, 0.2, 6/12, 0.4, 400, 5, 5/1200).

We try to gain best result by using finer grid:

[Call, Put] = blsprice (60, 60, 0.2, 6/12, 0.3)

Put = 2.4963;

PricingEurOptExpl (60, 60, 0.2, 6/12, 0.3, 400, 5, 5/1200)

ans = 2.3963

Again, noticed that the numerical method give us more accurate result but still can improve the result by using the more delicate grid:

PricingEurOptExpl (60, 60, 0.2, 6/12, 0.3, 400, 4, 5/1200)

ans = 2.4322;

PricingEurOptExpl (60, 60, 0.2, 6/12, 0.3, 400, 1, 5/1200)

ans = -1.5660e + 51

From previous example, conclude that explicit method leads to numerical instability. In order to solve this problem, implicit method was recommended to use to solve the instability.

[Call, Put] = blsprice (60, 60, 0.2, 6/12, 0.4)

Put = 3.9663

PricingEurOptImpl (60, 60, 0.2, 6/12, 0.4, 400, 5, 5/1200)

ans = 3.8870

The results of this scheme are approximate and also can be improved using more precise grid, without risk of numerical instability in the implementation.

1. **Water Flow**

* **Problem Statement**

Water is important for everyone. Although water is a renewable resource, water scarcity is occurred in the Floridan Aquifer which is one of the primary sources of ground-water in United States. According to Merwin (2014), the water level in Floridan Aquifer’s wells rose to the 75th percentile due to increasing rainfall and it only span for four months. After four months, the water level declined to the 50th percentile. The water levels of this aquifer need to maintain in order to keep the sustainability of procuring drinkable water for domestic utilization. Hence, the partial differential equation is used to analysis the behaviour and time evolution of the phenomenon and to observe the physical changes of the travelling-wave solution when the speed of the wave and time is increased incrementally.

* **Methodology**

In this study, the partial differential equation (PDE), which illustrates the flow of water under gravity through a homogeneous isotropic porous medium, is stated as:

PDE can be reduced to a solvable form which is ordinary differential equation (ODE) by changing the variable. The ODE after changing variable is:

The variable in this study is:

In United States, the water flow of Floridan Aquifer is discussed by using the tangent hyperbolic method in order to obtain the significant physical solution. The tangent hyperbolic method is based on the assumption that a travelling-wave solution can be expressed in terms of tangent x or tangent hyperbolic x. The solution after doing partial differential is:

(1)

Where is a constant, is speed, is time.

* **Results and Discussion**

Substitute the value of speed and time into equation (1). We get the results as below:

When ,

Plot them into a graph and get the figure 1 as below:

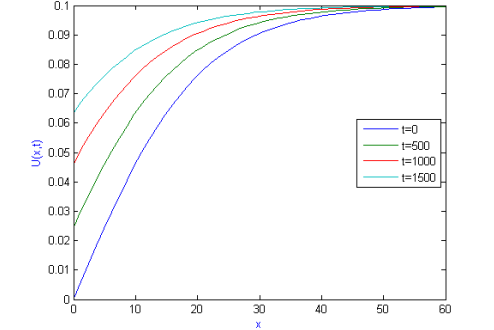


Figure 1: Plot of expression when at time *t*

The figure 1 shows that absolute convergence and as the time increases the amplitude of the waves also increases.

When ,

Plot them into graph and get the figure 2 as below:

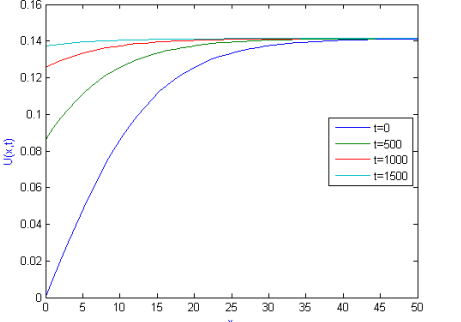


Figure 2: Plot of expression when at time *t*

The figure 2 shows that absolute convergence and as time increases the amplitude of the waves also increases.

When and

Plot them into graph and get the figure 3 as below:

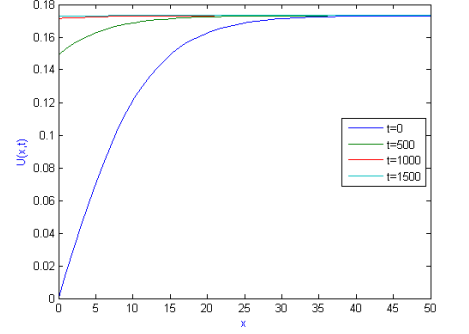


Figure 3: Plot of expression when at time *t*

The figure 3 shows that absolute convergence and as the time increases the amplitude of the waves also increases.

Since all of the three graphs show that convergence, we can conclude that as the speed of the tangent hyperbolic wave-like solution increases the greater the amplitude becomes. According to the Floridan Aquifer, we know that as the flow of water into the aquifer reduces the amount of water will not decay linearly; rather, we will see exponential decay.

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